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11e

RON LARSON BRUCE EDWARDS

Index of Applications

Engineering and Physical Sciences

(continued on back inside cover)

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11e

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***Calculus*, Eleventh Edition**
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Umbilic Torus by Helaman Ferguson, donated to Stony Brook University

The cover image is the Umbilic Torus statue created in 2012 by the famed sculptor and mathematician Dr. Helaman Ferguson. This statue weighs 10 tons and has a height of 24 feet. It is located at Stony Brook University in Stony Brook, New York.

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15 ▷ Vector Analysis 1043

16 ▷ Additional Topics in Differential Equations (Online)*

Appendices

*Available at the text-specific website *www.cengagebrain.com*

Preface

Welcome to *Calculus*, Eleventh Edition. We are excited to offer you a new edition with even more resources that will help you understand and master calculus. This textbook includes features and resources that continue to make *Calculus* a valuable learning tool for students and a trustworthy teaching tool for instructors.

Calculus provides the clear instruction, precise mathematics, and thorough coverage that you expect for your course. Additionally, this new edition provides you with **free** access to three companion websites:

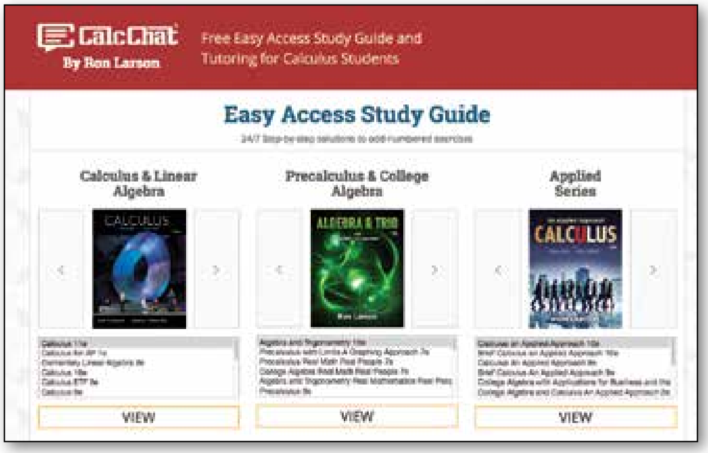
- **CalcView.com**—video solutions to selected exercises
- **CalcChat.com**—worked-out solutions to odd-numbered exercises and access to online tutors
- **LarsonCalculus.com**—companion website with resources to supplement your learning

These websites will help enhance and reinforce your understanding of the material presented in this text and prepare you for future mathematics courses. CalcView® and CalcChat® are also available as free mobile apps.

Features

NEW CalcView®

The website *CalcView.com* contains video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. The CalcView mobile app is available for free at the Apple® App Store® or Google Play™ store. The app features an embedded QR Code® reader that can be used to scan the on-page codes and go directly to the videos. You can also access the videos at CalcView.com.



UPDATED CalcChat®

In each exercise set, be sure to notice the reference to *CalcChat.com*. This website provides free step-by-step solutions to all odd-numbered exercises in many of our textbooks. Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. For over 14 years, hundreds of thousands of students have visited this site for help. The CalcChat mobile app is also available as a free download at the Apple® App Store® or Google Play™ store and features an embedded QR Code® reader.

App Store is a service mark of Apple Inc. Google Play is a trademark of Google Inc.
QR Code is a registered trademark of Denso Wave Incorporated.

REVISED LarsonCalculus.com

All companion website features have been updated based on this revision. Watch videos explaining concepts or proofs from the book, explore examples, view three-dimensional graphs, download articles from math journals, and much more.

NEW Conceptual Exercises

The *Concept Check* exercises and *Exploring Concepts* exercises appear in each section. These exercises will help you develop a deeper and clearer knowledge of calculus. Work through these exercises to build and strengthen your understanding of the calculus concepts and to prepare you for the rest of the section exercises.

REVISED Exercise Sets

The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant and to include topics our users have suggested. The exercises are organized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations.

REVISED Section Projects

Projects appear in selected sections and encourage you to explore applications related to the topics you are studying. We have added new projects, revised others, and kept some of our favorites. All of these projects provide an interesting and engaging way for you and other students to work and investigate ideas collaboratively.

Table of Contents Changes

Based on market research and feedback from users, we have made several changes to the table of contents.

- We added a review of trigonometric functions (Section P.4) to Chapter P.
- To cut back on the length of the text, we moved previous Section P.4 *Fitting Models to Data* (now Appendix G in the Eleventh Edition) to the text-specific website at *CengageBrain.com.*
- To provide more flexibility to the order of coverage of calculus topics, Section 3.5 *Limits at Infinity* was revised so that it can be covered after Section 1.5 *Infinite Limits*. As a result of this revision, some exercises moved from Section 3.5 to Section 3.6 *A Summary of Curve Sketching*.
- We moved Section 4.6 *Numerical Integration* to Section 8.6.
- We moved Section 8.7 *Indeterminate Forms and L'Hôpital's Rule* to Section 5.6.

Chapter Opener

Each Chapter Opener highlights real-life applications used in the examples and exercises.

Section Objectives

A bulleted list of learning objectives provides you with the opportunity to preview what will be presented in the upcoming section.

Theorems

Theorems provide the conceptual framework for calculus. Theorems are clearly stated and separated from the rest of the text by boxes for quick visual reference. Key proofs often follow the theorem and can be found at *LarsonCalculus.com.*

Definitions

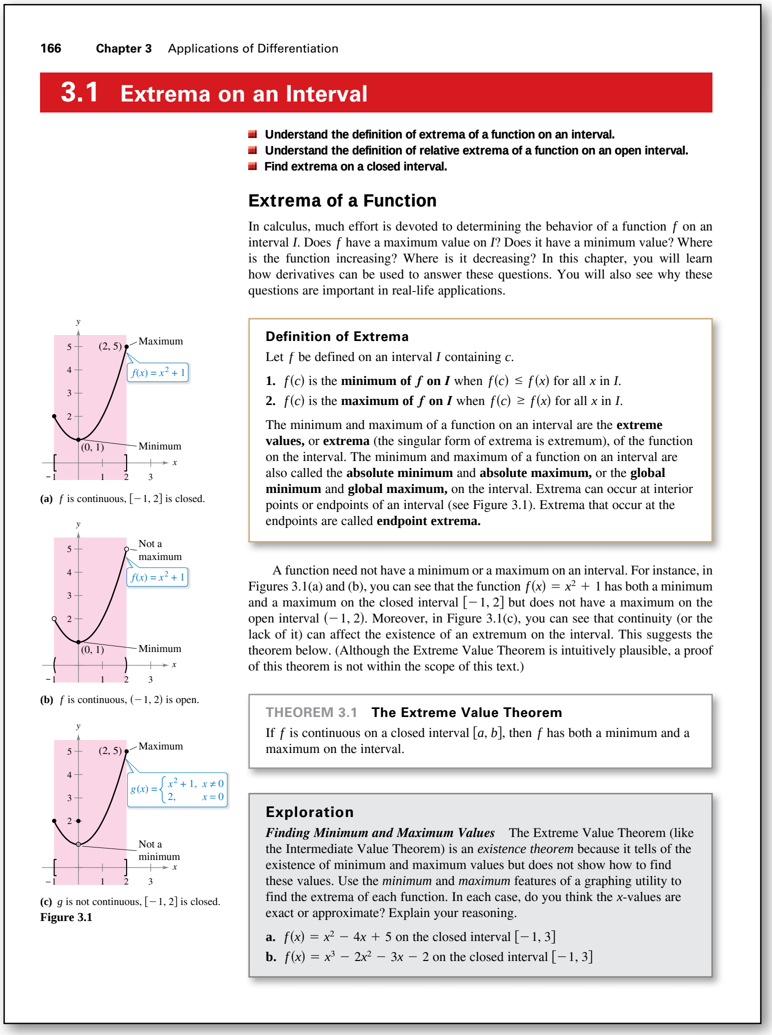
As with theorems, definitions are clearly stated using precise, formal wording and are separated from the text by boxes for quick visual reference.

Explorations

Explorations provide unique challenges to study concepts that have not yet been formally covered in the text. They allow you to learn by discovery and introduce topics related to ones presently being studied. Exploring topics in this way encourages you to think outside the box.

Remarks

These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show alternative or additional steps to a solution of an example.



166 **Chapter 3** Applications of Differentiation

3.1 Extrema on an Interval

- **Understand the definition of extrema of a function on an interval.**
- **Understand the definition of relative extrema of a function on an open interval.**
- **Find extrema on a closed interval.**

Extrema of a Function

In calculus, much effort is devoted to determining the behavior of a function f on an interval I. Does f have a maximum value on I? Does it have a minimum value? Where is the function increasing? Where is it decreasing? In this chapter, you will learn how derivatives can be used to answer these questions. You will also see why these questions are important in real-life applications.

Definition of Extrema

Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of f on I** when $f(c) \le f(x)$ for all x in I.
2. $f(c)$ is the **maximum of f on I** when $f(c) \ge f(x)$ for all x in I.

The minimum and maximum of a function on an interval are the **extreme values,** or **extrema** (the singular form of extrema is extremum), of the function on the interval. The minimum and maximum of a function on an interval are also called the **absolute minimum** and **absolute maximum,** or the **global minimum** and **global maximum,** on the interval. Extrema can occur at interior points or endpoints of an interval (see Figure 3.1). Extrema that occur at the endpoints are called **endpoint extrema.**

(a) f is continuous, $[-1, 2]$ is closed.

(b) f is continuous, $(-1, 2)$ is open.

(c) g is not continuous, $[-1, 2]$ is closed.

Figure 3.1

A function need not have a minimum or a maximum on an interval. For instance, in Figures 3.1(a) and (b), you can see that the function $f(x) = x^2 + 1$ has both a minimum and a maximum on the closed interval $[-1, 2]$ but does not have a maximum on the open interval $(-1, 2)$. Moreover, in Figure 3.1(c), you can see that continuity (or the lack of it) can affect the existence of an extremum on the interval. This suggests the theorem below. (Although the Extreme Value Theorem is intuitively plausible, a proof of this theorem is not within the scope of this text.)

THEOREM 3.1 The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

Exploration

Finding Minimum and Maximum Values The Extreme Value Theorem (like the Intermediate Value Theorem) is an *existence theorem* because it tells of the existence of minimum and maximum values but does not show how to find these values. Use the *minimum* and *maximum* features of a graphing utility to find the extrema of each function. In each case, do you think the x-values are exact or approximate? Explain your reasoning.

a. $f(x) = x^2 - 4x + 5$ on the closed interval $[-1, 3]$

b. $f(x) = x^3 - 2x^2 - 3x - 2$ on the closed interval $[-1, 3]$

How Do You See It? Exercise

The How Do You See It? exercise in each section presents a problem that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

Applications

Carefully chosen applied exercises and examples are included throughout to address the question, "When will I use this?" These applications are pulled from diverse sources, such as current events, world data, industry trends, and more, and relate to a wide range of interests. Understanding where calculus is (or can be) used promotes fuller understanding of the material.

Historical Notes and Biographies

Historical Notes provide you with background information on the foundations of calculus. The Biographies introduce you to the people who created and contributed to calculus.

Technology

Throughout the book, technology boxes show you how to use technology to solve problems and explore concepts of calculus. These tips also point out some pitfalls of using technology.

Putnam Exam Challenges

Putnam Exam questions appear in selected sections. These actual Putnam Exam questions will challenge you and push the limits of your understanding of calculus.

Student Resources

Student Solutions Manual for Calculus of a Single Variable
ISBN-13: 978-1-337-27538-5

Student Solutions Manual for Multivariable Calculus
ISBN-13: 978-1-337-27539-2

Need a leg up on your homework or help to prepare for an exam? The *Student Solutions Manuals* contain worked-out solutions for all odd-numbered exercises in *Calculus of a Single Variable* 11e (Chapters P–10 of *Calculus* 11e) and *Multivariable Calculus* 11e (Chapters 11–16 of *Calculus* 11e). These manuals are great resources to help you understand how to solve those tough problems.

CengageBrain.com
To access additional course materials, please visit *www.cengagebrain.com*. At the *CengageBrain.com* home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where these resources can be found.

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MindTap® provides you with the tools you need to better manage your limited time—you can complete assignments whenever and wherever you are ready to learn with course material specifically customized for you by your instructor and streamlined in one proven, easy-to-use interface. With an array of tools and apps—from note taking to flashcards—you'll get a true understanding of course concepts, helping you to achieve better grades and setting the groundwork for your future courses. This access code entitles you to 3 terms of usage.

Enhanced WebAssign®
Enhanced WebAssign (assigned by the instructor) provides you with instant feedback on homework assignments. This online homework system is easy to use and includes helpful links to textbook sections, video examples, and problem-specific tutorials.

Instructor Resources

Complete Solutions Manual for Calculus of a Single Variable, Vol. 1
ISBN-13: 978-1-337-27540-8

Complete Solutions Manual for Calculus of a Single Variable, Vol. 2
ISBN-13: 978-1-337-27541-5

Complete Solutions Manual for Multivariable Calculus
ISBN-13: 978-1-337-27542-2

The *Complete Solutions Manuals* contain worked-out solutions to all exercises in the text. They are posted on the instructor companion website.

Instructor's Resource Guide (on instructor companion site)
This robust manual contains an abundance of instructor resources keyed to the textbook at the section and chapter level, including section objectives, teaching tips, and chapter projects.

Cengage Learning Testing Powered by Cognero (login.cengage.com)
CLT is a flexible online system that allows you to author, edit, and manage test bank content; create multiple test versions in an instant; and deliver tests from your LMS, your classroom, or wherever you want. This is available online via *www.cengage.com/login.*

Instructor Companion Site
Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online via *www.cengage.com/login*. Access and download PowerPoint® presentations, images, instructor's manual, and more.

Test Bank (on instructor companion site)
The Test Bank contains text-specific multiple-choice and free-response test forms.

MindTap for Mathematics
MindTap® is the digital learning solution that helps you engage and transform today's students into critical thinkers. Through paths of dynamic assignments and applications that you can personalize, real-time course analytics, and an accessible reader, MindTap helps you turn cookie cutter into cutting edge, apathy into engagement, and memorizers into higher-level thinkers.

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Exclusively from Cengage Learning, Enhanced WebAssign combines the exceptional mathematics content that you know and love with the most powerful online homework solution, WebAssign. Enhanced WebAssign engages students with immediate feedback, rich tutorial content, and interactive, fully customizable e-books (YouBook), helping students to develop a deeper conceptual understanding of their subject matter. Quick Prep and Just In Time exercises provide opportunities for students to review prerequisite skills and content, both at the start of the course and at the beginning of each section. Flexible assignment options give instructors the ability to release assignments conditionally on the basis of students' prerequisite assignment scores. Visit us at **www.cengage.com/ewa** to learn more.

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On a personal level, we are grateful to our wives, Deanna Gilbert Larson and Consuelo Edwards, for their love, patience, and support. Also, a special note of thanks goes out to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write to us. Over the years we have received many useful comments from both instructors and students, and we value these very much.

Ron Larson

Bruce Edwards

P Preparation for Calculus

Automobile Aerodynamics *(Exercise 95, p. 30)*

Ferris Wheel
(Exercise 74, p. 40)

Conveyor Design *(Exercise 26, p. 16)*

Cell Phone Subscribers
(Exercise 68, p. 9)

Modeling Carbon Dioxide Concentration *(Example 6, p. 7)*

Clockwise from top left, iStockphoto.com/EdStock; DR-Media/Shutterstock.com;
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P.1 Graphs and Models

- **Sketch the graph of an equation.**
- **Find the intercepts of a graph.**
- **Test a graph for symmetry with respect to an axis and the origin.**
- **Find the points of intersection of two graphs.**
- **Interpret mathematical models for real-life data.**

RENÉ DESCARTES (1596–1650)

Descartes made many contributions to philosophy, science, and mathematics. The idea of representing points in the plane by pairs of real numbers and representing curves in the plane by equations was described by Descartes in his book *La Géométrie*, published in 1637.

See LarsonCalculus.com to read more of this biography.

The Graph of an Equation

In 1637, the French mathematician René Descartes revolutionized the study of mathematics by combining its two major fields—algebra and geometry. With Descartes's coordinate plane, geometric concepts could be formulated analytically and algebraic concepts could be viewed graphically. The power of this approach was such that within a century of its introduction, much of calculus had been developed.

The same approach can be followed in your study of calculus. That is, by viewing calculus from multiple perspectives—*graphically*, *analytically*, and *numerically*—you will increase your understanding of core concepts.

Consider the equation $3x + y = 7$. The point $(2, 1)$ is a **solution point** of the equation because the equation is satisfied (is true) when 2 is substituted for x and 1 is substituted for y. This equation has many other solutions, such as $(1, 4)$ and $(0, 7)$. To find other solutions systematically, solve the original equation for y.

$y = 7 - 3x$ Analytic approach

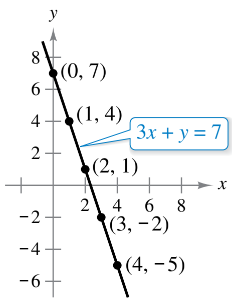
Then construct a **table of values** by substituting several values of x.

x	0	1	2	3	4
y	7	4	1	-2	-5

Numerical approach

From the table, you can see that $(0, 7)$, $(1, 4)$, $(2, 1)$, $(3, -2)$, and $(4, -5)$ are solutions of the original equation $3x + y = 7$. Like many equations, this equation has an infinite number of solutions. The set of all solution points is the **graph** of the equation, as shown in Figure P.1. Note that the sketch shown in Figure P.1 is referred to as the graph of $3x + y = 7$, even though it really represents only a *portion* of the graph. The entire graph would extend beyond the page.

In this course, you will study many sketching techniques. The simplest is point plotting—that is, you plot points until the basic shape of the graph seems apparent.

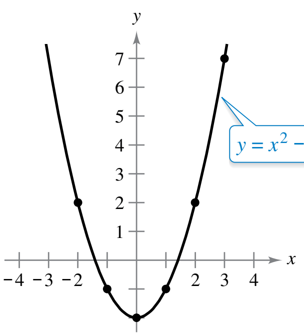


Graphical approach: $3x + y = 7$
Figure P.1

EXAMPLE 1 Sketching a Graph by Point Plotting

To sketch the graph of $y = x^2 - 2$, first construct a table of values. Next, plot the points shown in the table. Then connect the points with a smooth curve, as shown in Figure P.2. This graph is a **parabola.** It is one of the conics you will study in Chapter 10.

x	-2	-1	0	1	2	3
y	2	-1	-2	-1	2	7



The parabola $y = x^2 - 2$
Figure P.2

Granger, NYC

One disadvantage of point plotting is that to get a good idea about the shape of a graph, you may need to plot many points. With only a few points, you could badly misrepresent the graph. For instance, to sketch the graph of

$$y = \frac{1}{30}x(39 - 10x^2 + x^4)$$

you plot five points:

$$(-3, -3), \quad (-1, -1), \quad (0, 0), \quad (1, 1), \quad \text{and} \quad (3, 3)$$

as shown in Figure P.3(a). From these five points, you might conclude that the graph is a line. This, however, is not correct. By plotting several more points, you can see that the graph is more complicated, as shown in Figure P.3(b).

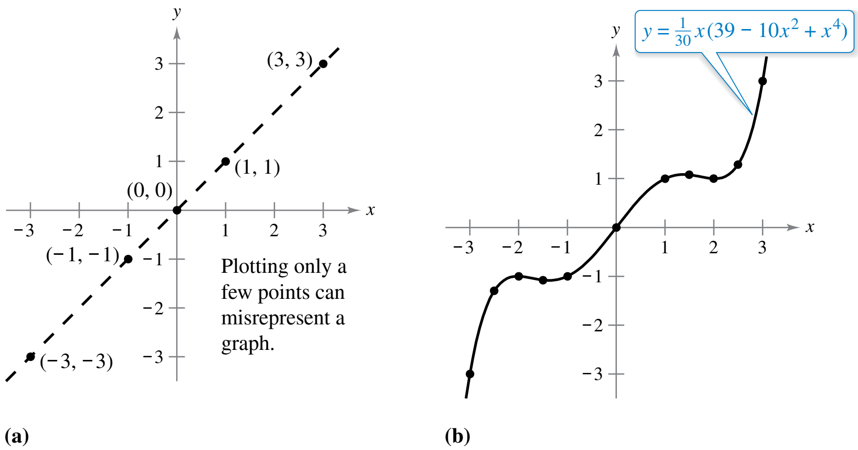


Figure P.3

Exploration

Comparing Graphical and Analytic Approaches

Use a graphing utility to graph each equation. In each case, find a viewing window that shows the important characteristics of the graph.

a. $y = x^3 - 3x^2 + 2x + 5$

b. $y = x^3 - 3x^2 + 2x + 25$

c. $y = -x^3 - 3x^2 + 20x + 5$

d. $y = 3x^3 - 40x^2 + 50x - 45$

e. $y = -(x + 12)^3$

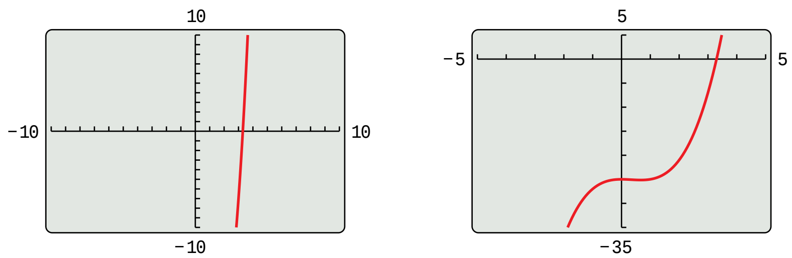
f. $y = (x - 2)(x - 4)(x - 6)$

A purely graphical approach to this problem would involve a simple "guess, check, and revise" strategy. What types of things do you think an analytic approach might involve? For instance, does the graph have symmetry? Does the graph have turns? If so, where are they? As you proceed through Chapters 1, 2, and 3 of this text, you will study many new analytic tools that will help you analyze graphs of equations such as these.

TECHNOLOGY Graphing an equation has been made easier by technology. Even with technology, however, it is possible to misrepresent a graph badly. For instance, each of the graphing utility* screens in Figure P.4 shows a portion of the graph of

$$y = x^3 - x^2 - 25.$$

From the screen on the left, you might assume that the graph is a line. From the screen on the right, however, you can see that the graph is not a line. So, whether you are sketching a graph by hand or using a graphing utility, you must realize that different "viewing windows" can produce very different views of a graph. In choosing a viewing window, your goal is to show a view of the graph that fits well in the context of the problem.



Graphing utility screens of $y = x^3 - x^2 - 25$
Figure P.4

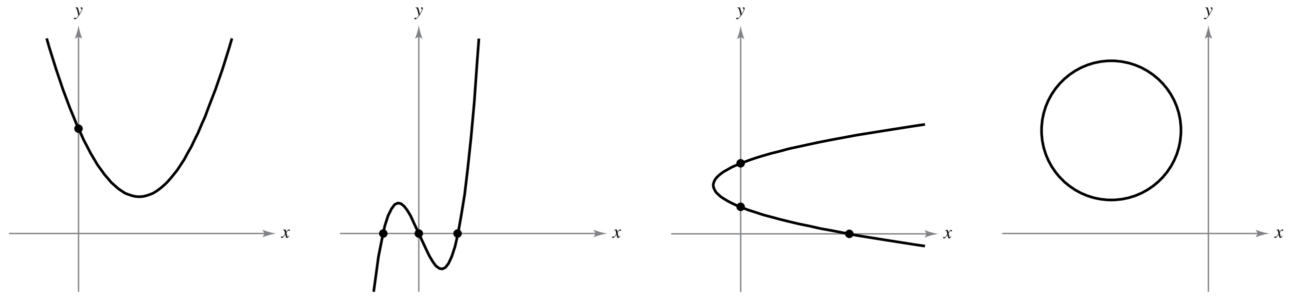
*In this text, the term *graphing utility* means either a graphing calculator, such as the *TI-Nspire*, or computer graphing software, such as *Maple* or *Mathematica*.

Intercepts of a Graph

REMARK Some texts denote the x-intercept as the x-coordinate of the point $(a, 0)$ rather than the point itself. Unless it is necessary to make a distinction, when the term *intercept* is used in this text, it will mean either the point or the coordinate.

Two types of solution points that are especially useful in graphing an equation are those having zero as their x- or y-coordinate. Such points are called **intercepts** because they are the points at which the graph intersects the x- or y-axis. The point $(a, 0)$ is an ***x*-intercept** of the graph of an equation when it is a solution point of the equation. To find the x-intercepts of a graph, let y be zero and solve the equation for x. The point $(0, b)$ is a ***y*-intercept** of the graph of an equation when it is a solution point of the equation. To find the y-intercepts of a graph, let x be zero and solve the equation for y.

It is possible for a graph to have no intercepts, or it might have several. For instance, consider the four graphs shown in Figure P.5.



No x-intercepts
One y-intercept

Three x-intercepts
One y-intercept

One x-intercept
Two y-intercepts

No intercepts

Figure P.5

EXAMPLE 2 Finding *x*- and *y*-Intercepts

Find the x- and y-intercepts of the graph of $y = x^3 - 4x$.

Solution To find the x-intercepts, let y be zero and solve for x.

$$x^3 - 4x = 0 \qquad \text{Let } y \text{ be zero.}$$

$$x(x - 2)(x + 2) = 0 \qquad \text{Factor.}$$

$$x = 0, 2, \text{ or } -2 \qquad \text{Solve for } x.$$

Because this equation has three solutions, you can conclude that the graph has three x-intercepts:

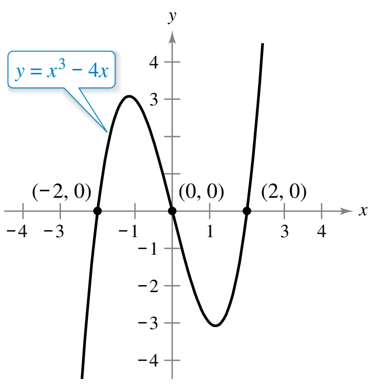
$$(0, 0), \quad (2, 0), \quad \text{and} \quad (-2, 0). \qquad x\text{-intercepts}$$

To find the y-intercepts, let x be zero. Doing this produces $y = 0$. So, the y-intercept is

$$(0, 0). \qquad y\text{-intercept}$$

(See Figure P.6.)

TECHNOLOGY Example 2 uses an analytic approach to finding intercepts. When an analytic approach is not possible, you can use a graphical approach by finding the points at which the graph intersects the axes. Use the *trace* feature of a graphing utility to approximate the intercepts of the graph of the equation in Example 2. Note that your utility may have a built-in program that can find the x-intercepts of a graph. (Your utility may call this the *root* or *zero* feature.) If so, use the program to find the x-intercepts of the graph of the equation in Example 2.



Intercepts of a graph
Figure P.6

Symmetry of a Graph

Knowing the symmetry of a graph before attempting to sketch it is useful because you need only half as many points to sketch the graph. The three types of symmetry listed below can be used to help sketch the graphs of equations (see Figure P.7).

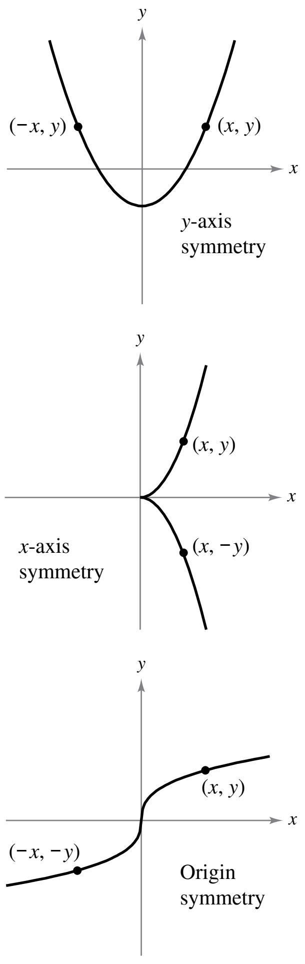


Figure P.7

1. A graph is **symmetric with respect to the *y*-axis** if, whenever (x, y) is a point on the graph, then $(-x, y)$ is also a point on the graph. This means that the portion of the graph to the left of the y-axis is a mirror image of the portion to the right of the y-axis.
2. A graph is **symmetric with respect to the *x*-axis** if, whenever (x, y) is a point on the graph, then $(x, -y)$ is also a point on the graph. This means that the portion of the graph below the x-axis is a mirror image of the portion above the x-axis.
3. A graph is **symmetric with respect to the origin** if, whenever (x, y) is a point on the graph, then $(-x, -y)$ is also a point on the graph. This means that the graph is unchanged by a rotation of 180° about the origin.

Tests for Symmetry

1. The graph of an equation in x and y is symmetric with respect to the y-axis when replacing x by $-x$ yields an equivalent equation.
2. The graph of an equation in x and y is symmetric with respect to the x-axis when replacing y by $-y$ yields an equivalent equation.
3. The graph of an equation in x and y is symmetric with respect to the origin when replacing x by $-x$ and y by $-y$ yields an equivalent equation.

The graph of a polynomial has symmetry with respect to the y-axis when each term has an even exponent (or is a constant). For instance, the graph of

$$y = 2x^4 - x^2 + 2$$

has symmetry with respect to the y-axis. Similarly, the graph of a polynomial has symmetry with respect to the origin when each term has an odd exponent, as illustrated in Example 3.

EXAMPLE 3 Testing for Symmetry

Test the graph of $y = 2x^3 - x$ for symmetry with respect to (a) the y-axis and (b) the origin.

Solution

a. $y = 2x^3 - x$ — Write original equation.

$y = 2(-x)^3 - (-x)$ — Replace x by $-x$.

$y = -2x^3 + x$ — Simplify. The result is *not* an equivalent equation.

Because replacing x by $-x$ does *not* yield an equivalent equation, you can conclude that the graph of $y = 2x^3 - x$ is *not* symmetric with respect to the y-axis.

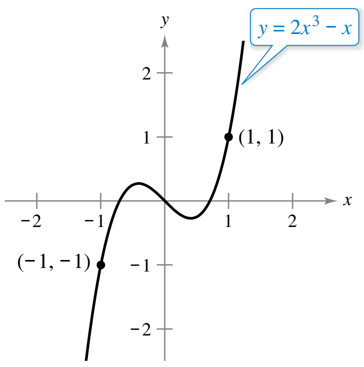
b. $y = 2x^3 - x$ — Write original equation.

$-y = 2(-x)^3 - (-x)$ — Replace x by $-x$ and y by $-y$.

$-y = -2x^3 + x$ — Simplify.

$y = 2x^3 - x$ — Equivalent equation

Because replacing x by $-x$ and y by $-y$ yields an equivalent equation, you can conclude that the graph of $y = 2x^3 - x$ is symmetric with respect to the origin, as shown in Figure P.8.



Origin symmetry
Figure P.8

EXAMPLE 4 Using Intercepts and Symmetry to Sketch a Graph

See LarsonCalculus.com for an interactive version of this type of example.

Sketch the graph of $x - y^2 = 1$.

Solution The graph is symmetric with respect to the x-axis because replacing y by $-y$ yields an equivalent equation.

$$x - y^2 = 1 \quad \text{Write original equation.}$$
$$x - (-y)^2 = 1 \quad \text{Replace } y \text{ by } -y.$$
$$x - y^2 = 1 \quad \text{Equivalent equation}$$

This means that the portion of the graph below the x-axis is a mirror image of the portion above the x-axis. To sketch the graph, first plot the x-intercept and the points above the x-axis. Then reflect in the x-axis to obtain the entire graph, as shown in Figure P.9.

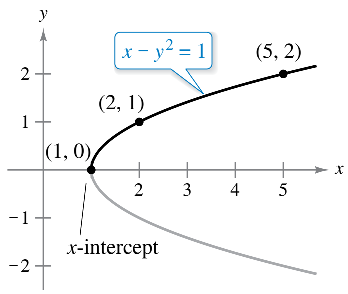


Figure P.9

TECHNOLOGY Graphing utilities are designed so that they most easily graph equations in which y is a function of x (see Section P.3 for a definition of *function*). To graph other types of equations, you need to split the graph into two or more parts *or* you need to use a different graphing mode. For instance, to graph the equation in Example 4, you can split it into two parts.

$$y_1 = \sqrt{x - 1} \quad \text{Top portion of graph}$$
$$y_2 = -\sqrt{x - 1} \quad \text{Bottom portion of graph}$$

Points of Intersection

A **point of intersection** of the graphs of two equations is a point that satisfies both equations. You can find the point(s) of intersection of two graphs by solving their equations simultaneously.

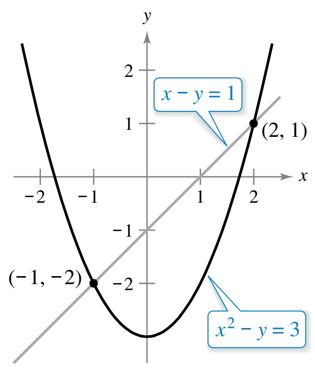
EXAMPLE 5 Finding Points of Intersection

Find all points of intersection of the graphs of

$$x^2 - y = 3 \quad \text{and} \quad x - y = 1.$$

Solution Begin by sketching the graphs of both equations in the *same* rectangular coordinate system, as shown in Figure P.10. From the figure, it appears that the graphs have two points of intersection. You can find these two points as follows.

$$y = x^2 - 3 \quad \text{Solve first equation for } y.$$
$$y = x - 1 \quad \text{Solve second equation for } y.$$
$$x^2 - 3 = x - 1 \quad \text{Equate } y\text{-values.}$$
$$x^2 - x - 2 = 0 \quad \text{Write in general form.}$$
$$(x - 2)(x + 1) = 0 \quad \text{Factor.}$$
$$x = 2 \text{ or } -1 \quad \text{Solve for } x.$$



Two points of intersection
Figure P.10

The corresponding values of y are obtained by substituting $x = 2$ and $x = -1$ into either of the original equations. Doing this produces two points of intersection:

$$(2, 1) \quad \text{and} \quad (-1, -2). \quad \text{Points of intersection}$$

You can check the points of intersection in Example 5 by substituting into *both* of the original equations or by using the *intersect* feature of a graphing utility.

Mathematical Models

Real-life applications of mathematics often use equations as **mathematical models.** In developing a mathematical model to represent actual data, you should strive for two (often conflicting) goals—accuracy and simplicity. That is, you want the model to be simple enough to be workable, yet accurate enough to produce meaningful results. Appendix G explores these goals more completely.

EXAMPLE 6 Comparing Two Mathematical Models

The Mauna Loa Observatory in Hawaii has been measuring the increasing concentration of carbon dioxide in Earth's atmosphere since 1958.

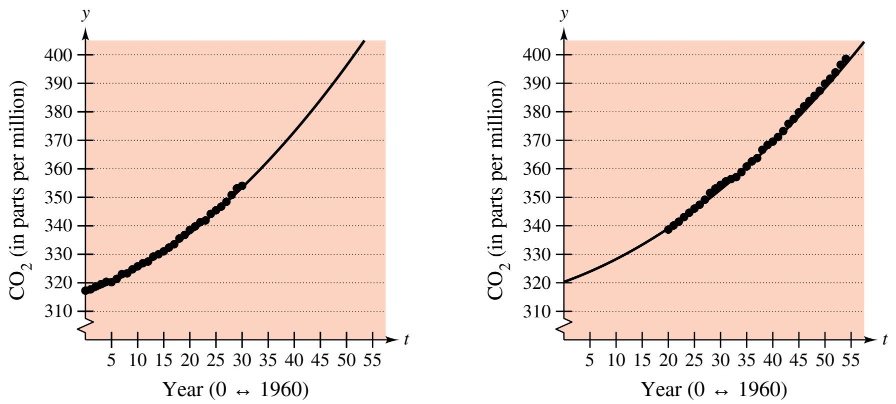
The Mauna Loa Observatory in Hawaii records the carbon dioxide concentration y (in parts per million) in Earth's atmosphere. The January readings for various years are shown in Figure P.11. In the July 1990 issue of *Scientific American*, these data were used to predict the carbon dioxide level in Earth's atmosphere in the year 2035, using the quadratic model

$$y = 0.018t^2 + 0.70t + 316.2 \quad \text{Quadratic model for 1960–1990 data}$$

where $t = 0$ represents 1960, as shown in Figure P.11(a). The data shown in Figure P.11(b) represent the years 1980 through 2014 and can be modeled by

$$y = 0.014t^2 + 0.66t + 320.3 \quad \text{Quadratic model for 1980–2014 data}$$

where $t = 0$ represents 1960. What was the prediction given in the *Scientific American* article in 1990? Given the second model for 1980 through 2014, does this prediction for the year 2035 seem accurate?



(a) **(b)**

Figure P.11

Solution To answer the first question, substitute $t = 75$ (for 2035) into the first model.

$$y = 0.018(75)^2 + 0.70(75) + 316.2 = 469.95 \quad \text{Model for 1960–1990 data}$$

So, the prediction in the *Scientific American* article was that the carbon dioxide concentration in Earth's atmosphere would reach about 470 parts per million in the year 2035. Using the model for the 1980–2014 data, the prediction for the year 2035 is

$$y = 0.014(75)^2 + 0.66(75) + 320.3 = 448.55. \quad \text{Model for 1980–2014 data}$$

So, based on the model for 1980–2014, it appears that the 1990 prediction was too high.

The models in Example 6 were developed using a procedure called *least squares regression* (see Section 13.9). The older model has a correlation of $r^2 \approx 0.997$, and for the newer model it is $r^2 \approx 0.999$. The closer r^2 is to 1, the "better" the model.

P.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

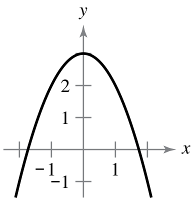
CONCEPT CHECK

1. **Finding Intercepts** Describe how to find the x- and y-intercepts of the graph of an equation.

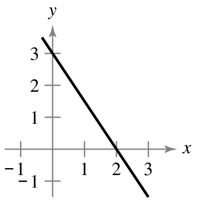
2. **Verifying Points of Intersection** How can you check that an ordered pair is a point of intersection of two graphs?

Matching **In Exercises 3–6, match the equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]**

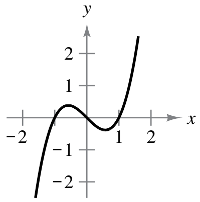
(a)



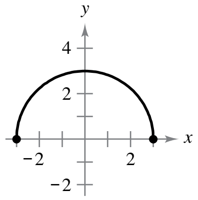
(b)



(c)



(d)



3. $y = -\frac{3}{2}x + 3$

4. $y = \sqrt{9 - x^2}$

5. $y = 3 - x^2$

6. $y = x^3 - x$

Sketching a Graph by Point Plotting **In Exercises 7–16, sketch the graph of the equation by point plotting.**

7. $y = \frac{1}{2}x + 2$

8. $y = 5 - 2x$

9. $y = 4 - x^2$

10. $y = (x - 3)^2$

11. $y = |x + 1|$

12. $y = |x| - 1$

13. $y = \sqrt{x} - 6$

14. $y = \sqrt{x + 2}$

15. $y = \dfrac{3}{x}$

16. $y = \dfrac{1}{x + 2}$

Approximating Solution Points Using Technology **In Exercises 17 and 18, use a graphing utility to graph the equation. Move the cursor along the curve to approximate the unknown coordinate of each solution point accurate to two decimal places.**

17. $y = \sqrt{5 - x}$
 (a) $(2, y)$
 (b) $(x, 3)$

18. $y = x^5 - 5x$
 (a) $(-0.5, y)$
 (b) $(x, -4)$

Finding Intercepts **In Exercises 19–28, find any intercepts.**

19. $y = 2x - 5$

20. $y = 4x^2 + 3$

21. $y = x^2 + x - 2$

22. $y^2 = x^3 - 4x$

23. $y = x\sqrt{16 - x^2}$

24. $y = (x - 1)\sqrt{x^2 + 1}$

25. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

26. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

27. $x^2y - x^2 + 4y = 0$

28. $y = 2x - \sqrt{x^2 + 1}$

Testing for Symmetry **In Exercises 29–40, test for symmetry with respect to each axis and to the origin.**

29. $y = x^2 - 6$

30. $y = 9x - x^2$

31. $y^2 = x^3 - 8x$

32. $y = x^3 + x$

33. $xy = 4$

34. $xy^2 = -10$

35. $y = 4 - \sqrt{x + 3}$

36. $xy - \sqrt{4 - x^2} = 0$

37. $y = \dfrac{x}{x^2 + 1}$

38. $y = \dfrac{x^5}{4 - x^2}$

39. $y = |x^3 + x|$

40. $|y| - x = 3$

Using Intercepts and Symmetry to Sketch a Graph **In Exercises 41–56, find any intercepts and test for symmetry. Then sketch the graph of the equation.**

41. $y = 2 - 3x$

42. $y = \frac{2}{3}x + 1$

43. $y = 9 - x^2$

44. $y = 2x^2 + x$

45. $y = x^3 + 2$

46. $y = x^3 - 4x$

47. $y = x\sqrt{x + 5}$

48. $y = \sqrt{25 - x^2}$

49. $x = y^3$

50. $x = y^4 - 16$

51. $y = \dfrac{8}{x}$

52. $y = \dfrac{10}{x^2 + 1}$

53. $y = 6 - |x|$

54. $y = |6 - x|$

55. $3y^2 - x = 9$

56. $x^2 + 4y^2 = 4$

Finding Points of Intersection **In Exercises 57–62, find the points of intersection of the graphs of the equations.**

57. $x + y = 8$
 $4x - y = 7$

58. $3x - 2y = -4$
 $4x + 2y = -10$

59. $x^2 + y = 15$
 $-3x + y = 11$

60. $x = 3 - y^2$
 $y = x - 1$

The symbol and a red exercise number indicates that a video solution can be seen at *CalcView.com*.

The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by the use of appropriate technology.

61. $x^2 + y^2 = 5$
$x - y = 1$

62. $x^2 + y^2 = 16$
$x + 2y = 4$

Finding Points of Intersection Using Technology **In Exercises 63–66, use a graphing utility to find the points of intersection of the graphs of the equations. Check your results analytically.**

63. $y = x^3 - 2x^2 + x - 1$
$y = -x^2 + 3x - 1$

64. $y = x^4 - 2x^2 + 1$
$y = 1 - x^2$

65. $y = \sqrt{x + 6}$
$y = \sqrt{-x^2 - 4x}$

66. $y = -|2x - 3| + 6$
$y = 6 - x$

67. Modeling Data The table shows the Gross Domestic Product, or GDP (in trillions of dollars), for 2009 through 2014. *(Source: U.S. Bureau of Economic Analysis)*

Year	2009	2010	2011	2012	2013	2014
GDP	14.4	15.0	15.5	16.2	16.7	17.3

(a) Use the regression capabilities of a graphing utility to find a mathematical model of the form $y = at + b$ for the data. In the model, y represents the GDP (in trillions of dollars) and t represents the year, with $t = 9$ corresponding to 2009.

(b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.

(c) Use the model to predict the GDP in the year 2024.

68. Modeling Data

The table shows the numbers of cell phone subscribers (in millions) in the United States for selected years. *(Source: CTIA-The Wireless Association)*

Year	2000	2002	2004	2006
Number	109	141	182	233

Year	2008	2010	2012	2014
Number	270	296	326	355

(a) Use the regression capabilities of a graphing utility to find a mathematical model of the form $y = at^2 + bt + c$ for the data. In the model, y represents the number of subscribers (in millions) and t represents the year, with $t = 0$ corresponding to 2000.

(b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.

(c) Use the model to predict the number of cell phone subscribers in the United States in the year 2024.

69. Break-Even Point Find the sales necessary to break even $(R = C)$ when the cost C of producing x units is $C = 2.04x + 5600$ and the revenue R from selling x units is $R = 3.29x$.

70. Using Solution Points For what values of k does the graph of $y^2 = 4kx$ pass through the point?

(a) $(1, 1)$ (b) $(2, 4)$

(c) $(0, 0)$ (d) $(3, 3)$

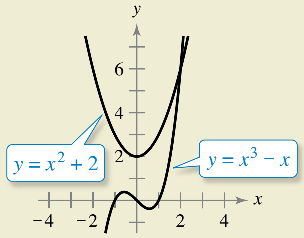
EXPLORING CONCEPTS

71. Using Intercepts Write an equation whose graph has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$. (There is more than one correct answer.)

72. Symmetry A graph is symmetric with respect to the x-axis and to the y-axis. Is the graph also symmetric with respect to the origin? Explain.

73. Symmetry A graph is symmetric with respect to one axis and to the origin. Is the graph also symmetric with respect to the other axis? Explain.

74. HOW DO YOU SEE IT? Use the graphs of the two equations to answer the questions below.



(a) What are the intercepts for each equation?

(b) Determine the symmetry for each equation.

(c) Determine the point of intersection of the two equations.

True or False? **In Exercises 75–78, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

75. If $(-4, -5)$ is a point on a graph that is symmetric with respect to the x-axis, then $(4, -5)$ is also a point on the graph.

76. If $(-4, -5)$ is a point on a graph that is symmetric with respect to the y-axis, then $(4, -5)$ is also a point on the graph.

77. If $b^2 - 4ac > 0$ and $a \neq 0$, then the graph of

$$y = ax^2 + bx + c$$

has two x-intercepts.

78. If $b^2 - 4ac = 0$ and $a \neq 0$, then the graph of

$$y = ax^2 + bx + c$$

has only one x-intercept.